

# C.U.SHAH UNIVERSITY

## Summer Examination-2017

**Subject Name : Engineering Mathematics-II**

**Subject Code : 4TE02EMT2**

**Branch: B.Tech (All)**

**Semester : 2**

**Date :04/05/2017**

**Time : 02:00 To 05:00**

**Marks : 70**

**Instructions:**

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q.1 Attempt the following questions:**

**(14)**

a)  $\int_0^{\frac{\pi}{2}} \cos^7 \theta d\theta = \underline{\hspace{2cm}}$ .

(a) 0      (b)  $\frac{16}{35}$       (c)  $\frac{32}{35}$       (d)  $\frac{8\pi}{35}$ .

b) If  $f_n = \int_0^{\frac{\pi}{4}} \tan^n \theta d\theta$  then  $f_n + f_{n-2} = \underline{\hspace{2cm}}$ .

(a)  $\frac{1}{n}$       (b)  $\frac{1}{n-1}$       (c)  $\frac{n}{n-1}$       (d)  $\frac{n-1}{n}$ .

c) State the Euler's formula.

d)  $\Gamma(4.5) = \underline{\hspace{2cm}}$ .

(a)  $4.5 \Gamma(3.5)$       (b)  $3.5 \Gamma(4)$       (c)  $4 \Gamma(3.5)$       (d) none of these.

e)  $\beta\left(\frac{1}{2}, \frac{1}{2}\right) = \underline{\hspace{2cm}}$ .

(a)  $\sqrt{\pi}$       (b)  $\frac{1}{2}$       (c) 1      (d)  $\pi$ .

f) Define a complete elliptical integral of 1<sup>st</sup> kind.

g)  $\operatorname{erf}(x) + \operatorname{erfc}(x) = \underline{\hspace{2cm}}$ .

(a) 0      (b) 1      (c) 2      (d) none of these.

h) The curve  $y^2(2a-x) = x^3$  is symmetrical about \_\_\_\_\_.

(a) X-axis      (b) Y-axis      (c) origin      (d) line  $y = x$ .

i) The length of the spiral  $r = e^{\theta/\sqrt{2}}$ ,  $0 \leq \theta \leq \pi$ , is \_\_\_\_\_.

(a)  $e^{\pi}$       (b)  $e^{\pi} + 1$       (c)  $e^{\pi} - 1$       (d) 1.

j)  $\int_0^{\pi} \int_0^x x \sin y dy dx = \underline{\hspace{2cm}}$ .

(a)  $\pi/2$       (b)  $\pi^2/2$       (c)  $(\pi/2) + 1$       (d)  $(\pi^2/2) + 1$ .



- k)  $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) dz dy dx = \underline{\hspace{2cm}}$ .  
 (a) 1      (b) -1      (c) 0      (d) none of these.
- l) The series  $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$  is \_\_\_\_\_.  
 (a) convergent      (b) divergent      (c) conditionally convergent  
 (d) none of these.
- m) The series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  is divergent if \_\_\_\_\_.  
 (a)  $p > 1$       (b)  $p \leq 1$       (c)  $p \geq 1$       (d)  $p < 1$ .
- n) The differential equation  $(x + y) dy + (x - y) dx = 0$  is \_\_\_\_\_ differential equation.

**Attempt any four questions from Q-2 to Q-8**

**Q.2 Attempt all questions** **(14)**

a) Evaluate: **(02)**

$$\int_0^{\pi} (1 + \cos \theta)^4 d\theta$$

b) Evaluate: **(02)**

$$\int_0^{\frac{1}{2}} x^3 \sqrt{1 - 4x^2} dx$$

c) Evaluate: **(04)**

$$\int_0^1 \frac{x^6}{(1 + x^2)} dx$$

d) Trace the curve  $x^3 + y^3 = 3axy$ . **(06)**

**Q.3 Attempt all questions** **(14)**

a) If  $\beta(n, 3) = 1/105$  and  $n$  is a positive integer, then find  $n$ . **(02)**

b) Prove that: (i)  $n\beta(m + 1, n) = m\beta(m, n + 1)$ ; and **(04)**  
 (ii)  $\beta(m, n) = \beta(m, n + 1) + \beta(m + 1, n)$ .

c) Evaluate: **(04)**

$$\int_{-\infty}^{\infty} e^{-k^2 x^2} dx$$

d) Prove that: **(04)**

$$\int_0^{\infty} \frac{x^4}{4^x} dx = \frac{24}{(\log 4)^5}$$



**Q.4 Attempt all questions** (14)

a) Prove that:

$$\int_0^{\infty} \frac{\sqrt{x}}{x^2 + 2x + 1} dx = \frac{\pi}{2} \quad (03)$$

b) Evaluate:

$$\int_0^{\frac{\pi}{3}} \frac{d\theta}{\sqrt{3 - 4\sin^2\theta}} \quad (03)$$

c) If the perimeter of the ellipse of  $e = 1/\sqrt{2}$  is equal to twice the length of one arch of the curve  $y = \sin x$ , then find the area of the ellipse. (04)

d) (1) Show that  $\operatorname{erf}(x)$  is an odd function; and (04)

(2) prove that:

$$\int_{-a}^a e^{-t^2} dt = \sqrt{\pi} \operatorname{erf}(a)$$

**Q.5 Attempt all questions** (14)

a) Trace the curve  $r^2 = a^2 \cos 2\theta$ . (04)

b) Find the length of the Cardioid  $r = 1 + \cos \theta$ . (02)

c) Find the area of the smaller region lying above X-axis and bounded by the circle  $x^2 + y^2 = 2x$  and the parabola  $y^2 = x$ . (04)

d) Find volume of the solid generated by revolving the lemniscate  $r^2 = a^2 \cos 2\theta$  about the line  $\theta = \pi/2$ . (04)

**Q.6 Attempt all questions** (14)

a) Evaluate:

$$\iint_R (x^2 + y^2) dA \quad (02)$$

, where R is a triangular region with vertices (0, 0), (0, 1) and (1, 0).

b) Evaluate: (03)

$$\int_0^1 \int_y^1 x^2 e^{xy} dx dy$$

c) Evaluate: (03)

$$\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2 + y^2 + 1) dx dy$$

d) Find the volume of the region D between the cylinder  $z = y^2$ , and the XY-plane that is bounded by the planes  $x = 0$ ,  $x = 1$ ,  $y = -1$ ,  $y = 1$ . (03)

e) Evaluate: (03)

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{\ln 3} \frac{\pi e^{2x} \sin(\pi y^2)}{y^2} dx dy dz$$



**Q.7 Attempt all questions** (14)

a) Discuss convergence/divergence of the following series:

i)  $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$     ii)  $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n+3}\sqrt[3]{n}}$     iii)  $\sum_{n=1}^{\infty} \frac{n^{10}}{10^n}$     iv)  $\sum_{n=1}^{\infty} (-1)^n \ln\left(1 + \frac{1}{n}\right)$  (08)

b) Prove that if the series  $\sum a_n$  converges, then (02)

$$\lim_{n \rightarrow \infty} a_n = 0$$

Find the values of x for which the following power series converges. (04)

c)  $\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n}$

**Q.8 Solve the following ordinary differential equations:** (14)

a)  $(x - y)dx - (x + y)dy = 0$  (03)

b)  $(y - x^3)dx + (x + y^3)dy = 0$  (03)

c)  $(1 + x^2)dy + 2xy dx = \cot x dx$  (04)

d)  $2yy'' = 1 + (y')^2$  (04)

